# An Extrinsic-Inductance Independent Approach for Direct Extraction of HBT Intrinsic Circuit Parameters

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***Abstract* - A new analytical procedure has been developed for direct extraction of the intrinsic elements in a hybrid-® equivalent circuit of heterojunction bipolar transistors (HBTs). The method differs from previous ones by formulating impedance-parameter based expressions that are exclusive of the extrinsic parasitic inductances associated with the base, emitter and collector. It is therefore not susceptible to variation of the extrinsic reactances from DC to high frequencies and can lead to very accurate extraction of the intrinsic elements under different bias conditions.**

## I. INTRODUCTION

In consideration of efficiency and uniqueness, direct extraction approaches have been intensively used for modeling HBTs [1]-[6]. All approaches without exception follow that the extrinsic elements are first determined and then de-embedded to allow for extraction of the intrinsic elements. The reported extraction methods for extrinsic elements rely on either test structure measurements [1] or forward-biased measurements under high base currents [2]-[6]. However, in extracting extrinsic inductances there are still some empirical difficulties in both techniques. The former technique requires separate measurement of S parameters in a shorted test structure with an identical interconnection pattern in a real device. To characterize the parasitic inductances effectively, the interconnection pattern is usually designed simply but not practically enough for modeling multiple HBTs in an array form which occurs frequently in the power amplifier applications. The latter technique takes the S-parameter measurement when HBT's base-collector and base-emitter junctions are maintained forward-biased. The injected base current densities have to be sufficiently high so as to make the junction impedances negligible when compared to the extrinsic impedances. Under this scheme the extrinsic inductances are better extracted with larger reactances at higher frequencies. Nevertheless, the range of base-current density and frequency for a robust extraction of extrinsic inductances can't normally be forecast unless intensive tests have been carried out for a while. To save strength in extracting extrinsic inductances but still have reliable extracted quantities for intrinsic elements, our formalism is quite unique to be able to express each intrinsic element in a mathematical form independent of all extrinsic inductances. In [6] an extrinsic-inductance insensitive expression for extracting base resistance has been proposed as a starting point for extracting the intrinsic elements. The entire formulation is still extrinsic-inductance dependent and at least needs some rough figures for all extrinsic inductances before extracting the intrinsic elements.

## II. FORMULATION

The two-port impedance parameters associated with the hybrid-® equivalent circuit shown in Fig. 1 for an HBT are derived as [6]

$$Z_{11} = j\mu(L_b + L_e) + R_e + \frac{R_b Z_{jx}}{R_b + Z_{bc} + Z_{jx}} + \frac{Z_{bc} R_b}{(1+GZ_{be})(R_b + Z_{bc} + Z_{jx})} + \frac{Z_{be}}{1+GZ_{be}}, \quad (1)$$

$$Z_{12} = j\mu L_e + R_e + \frac{Z_{bc} R_b}{(1+GZ_{be})(R_b + Z_{bc} + Z_{jx})} + \frac{Z_{be}}{1+GZ_{be}}, \quad (2)$$

$$Z_{21} = j\mu L_e + R_e + \frac{Z_{bc} R_b - GZ_{be} Z_{bc} Z_{jx}}{(1+GZ_{be})(R_b + Z_{bc} + Z_{jx})} + \frac{Z_{be}}{1+GZ_{be}}, \quad (3)$$

$$Z_{22} = j\mu(L_e + L_c) + R_e + R_c + \frac{Z_{bc}(R_b + Z_{jx})}{(1+GZ_{be})(R_b + Z_{bc} + Z_{jx})} + \frac{Z_{be}}{1+GZ_{be}}, \quad (4)$$

where $Z_{be} = (g_{be} + j\mu C_{be})^{-1}, Z_{bc} = (g_{bc} + j\mu C_{bc})^{-1}, Z_{jx} = (j\mu C_{jx})^{-1}$, and $G = g_m e^{-j\mu^2}$. We rearrange the formulation to find the extrinsic-inductance independent expressions given as

$$Z_{12} - Z_{21} = \frac{Z_{bc} Z_{jx}}{(1+U)(R_b + Z_{bc} + Z_{jx})}, \quad (5)$$

$$real\{Z_{11} - Z_{12}\} = real\left\{\frac{R_b Z_{jx}}{R_b + Z_{bc} + Z_{jx}}\right\}, \quad (6)$$

$$real\{Z_{22} - Z_{12}\} = R_c + real\{(Z_{12} - Z_{21})U\}, \quad (7)$$

$$real\{Z_{12}\} = R_e + real\left\{\frac{1}{G(1+U)}\right\} + real\left\{(Z_{12} - Z_{21})U\frac{R_b}{Z_{jx}}\right\}, \quad (8)$$

where

$$U = (GZ_{be})^{-1} = ( \;^{-1} + j\mu \frac{C_{be}}{g_m}) e^{j\mu^2}. \quad (9)$$

In (9) $\beta = g_m R_{be}$ and $e^{j\mu\tau}$ can be approximated by the Taylor expansion:

$$e^{j\mu\tau} \approx 1 + j\mu\tau - \frac{\mu^2\tau^2}{2}. \qquad (10)$$

After substituting (9) and (10) into (5), we inspect and then omit some terms that have negligible contribution based on the assumptions: $C_{be} >> \tau g_{be}$ , $C'_{bc} = C_{bc} + C_{jx} >> \tau g_{bc}$ and $C_{be} / C'_{bc} >> g_m R_b$. The equation in (5) can be approximated as

$$g_m^{-1}(Z_{12} - Z_{21})^{-1} \approx \left[\frac{g_{bc}}{g_m}(1+\beta^{-1}) - \frac{C_{be}C'_{bc}}{g_m^2}(\mu^2 - \frac{\mu^4\tau^2}{2})\right] + j\mu\left[\frac{C'_{bc}}{g_m}(1+\beta^{-1}) + \frac{C_{be}}{g_m}(\frac{g_{bc}}{g_m} - \mu^2\tau\frac{C'_{bc}}{g_m})\right]. \qquad (11)$$

Note that the above assumptions are always good for an HBT operating in the forward active region. The most crucial step in this extraction algorithm is to derive the phase of the difference between two mutual impedance parameters as

$$\cot\angle(Z_{12} - Z_{21}) \approx \frac{\mu^2 - \mu_0^2 - \frac{\mu^4\tau^2}{2}}{\mu k - \mu^3\tau}, \qquad (12)$$

where

$$\mu_0 = \sqrt{(1+\beta^{-1})\frac{g_{bc}g_m}{C_{be}C'_{bc}}}, \qquad (13)$$

and

$$k = (1+\beta^{-1})\frac{g_m}{C_{be}} + \frac{g_{bc}}{C'_{bc}}. \qquad (14)$$

Except for independence of extrinsic inductances, the use of such a phase term has another unique advantage. From an inspection of the intrinsic elements on the right hand side of (13) and (14), one can find that in the forward active region only $g_m$ and $C_{be}$ have exponential dependence on the base-emitter voltage $V_{be}$. The other parameters are much less dependent on $V_{be}$. An attraction of this formulation is that both $\mu_0$ and $k$ can be expressed in terms of the ratio $g_m / C_{be}$ that is also insensitive to $V_{be}$. This makes the proposed model extraction more stable to deal with the measurement data with noise due to fluctuation of the supply voltage.

At first, the measured S parameters are converted into the impedance parameters in which the cotangent of the phase angle for the difference between the mutual terms is calculated. Then the parameters $\mu_0$ and $k$ can be found by estimating the root and slope of the cotangent function respectively at low frequencies, whereas the delay time $\tau$ can be pin-pointed by a match of the curvature at high frequencies, as illustrated in Fig. 2 with a real example. Once $\mu_0$ and $k$ have been found, solving for $\tau$ in (12) can be regarded as a better way to determine $\tau$, which yields

$$\tau \approx \frac{\cot\angle(Z_{12} - Z_{21})}{\mu} - \frac{1}{\mu}\sqrt{\cot^2\angle(Z_{12} - Z_{21}) - \frac{2k}{\mu}\cot\angle(Z_{12} - Z_{21}) - 2(\frac{\mu_0^2}{\mu^2} - 1)}. \qquad (15)$$

Substituting (13),(14) into (5)~(8) and (11), we can further express all the intrinsic elements in terms of $\mu_0$, $k$, $\tau$ and impedance parameters as

$$C'_{bc} \approx real\{(Z_{12} - Z_{21})(1+\beta^{-1})[Q(\mu_0 - \frac{\mu^2}{\mu_0} + \frac{\mu^4\tau^2}{2\mu_0}) + j\mu(1 + Q^2 - \frac{Q\mu^2\tau}{\mu_0})]\}^{-1}, \qquad (16)$$

$$R_{bc} \approx (\mu_0 Q C'_{bc})^{-1}, \qquad (17)$$

$$R_b = \frac{real\{Z_{11} - Z_{12}\}}{real\{(Z_{12} - Z_{21})(1+U)(\mu_0 Q + j\mu X_{CJC})C'_{bc}\}}, \qquad (18)$$

$$g_m \approx \frac{real\{e^{j\mu\tau}(1+U)^{-1}\}}{real\{Z_{12}\} - R_e + \mu R_b(1 - X_{CJC})C'_{bc}imag\{U(Z_{12} - Z_{21})\}}, \qquad (19)$$

$$C_{be} \approx g_m(1+\beta^{-1})\frac{Q}{\mu_0}, \qquad (20)$$

$$R_{be} \approx \frac{\beta}{g_m}, \qquad (21)$$

where

$$U = \left[\beta^{-1} + j\mu\mu_0^{-1}Q(1+\beta^{-1})\right] e^{j\mu\tau}, \qquad (22)$$

$$Q = \frac{1 - \sqrt{1 - (2\mu_0 k^{-1})^2}}{2\mu_0 k^{-1}}, \qquad (23)$$

and

$$X_{CJC} = \frac{C_{bc}}{C'_{bc}} = 1 - \frac{C_{jx}}{C'_{bc}}. \qquad (24)$$

To determine all the intrinsic elements, the value of current gain $\beta$ selected can be either the static result or the magnitude of $H_{21}$ parameter at low frequencies. The emitter resistance $R_e$ can be found using the static flyback method. The ratio $X_{CJC}$ describes the distributed phenomena in the base region and can be estimated from the HBT geometry. With the above expedience, the extracted quantities of the intrinsic elements using (16)~(21) yield moderately accurate results that are good enough to support extraction of the extrinsic elements in the next step. By reexamining (1)~(4), the extrinsic elements can be expressed as

$$Z_c = R_c + j\mu L_c = (Z_{22} - Z_{12}) - U(Z_{12} - Z_{21}), \qquad (25)$$

$$\mu L_b = imag\{(Z_{11} - Z_{12}) - R_b C'_{bc}(Z_{12} - Z_{21})(1+U)(\mu_0 Q + j\mu X_{CJC})\}, \qquad (26)$$

$$\mu L_e = imag\{Z_{12} - j\mu R_b C'_{bc}(1 - X_{CJC})U(Z_{12} - Z_{21}) - [g_m e^{-j\mu\tau}(1+U)]^{-1}\}. \qquad (27)$$

These extrinsic elements characterize the passive contact structures and are assumed to be independent of bias conditions. Therefore, we can fully exploit their bias independence to modify the previously selected quantities for , $R_e$ and $X_{CJC}$ slightly and subsequently re-extract all the intrinsic elements under different bias conditions.

## III. Results

An n-p-n AlGaAs/GaAs HBT with an emitter area of 2.4x3 $\mu m^2$ has been modeled as an example. The HBT is biased in the common-emitter forward-active region with the collector-emitter voltage $V_{ce} = 3V$ and the collector current $I_c = 1.8mA$ Fig. 2 illustrates determination of the parameters $\mu_0, k$ and $\tau$ by performing a curve fit on the cotangent of the phase $\angle(Z_{12} - Z_{21})$. The estimated quantities for , $R_e$ and $X_{CJC}$ are 61.3, 29.9 $\Omega$ and 0.85 respectively. As first, the intrinsic elements including $g_m, R_{be}, C_{be}, C^t_{bc}, R_{bc}$ and $R_b$ were calculated from (16)-(21) and shown in Figs. 3~5 against frequency up to 26.5 GHz. It can be seen that the quantities are mostly frequency independent, which implies a strong degree of accuracy and robustness for this technique. With knowledge of the intrinsic elements, the extrinsic elements including $R_c, L_c, L_b$ and $L_e$ can be further evaluated using (25)-(27) with results demonstrated in Figs. 6 and 7. Finally, the S parameters were simulated based on the equivalent model in Fig. 1 and compared to the measured data, as shown in Fig. 8. Excellent agreement has been found.

## IV. Conclusion

An extrinsic-inductance independent approach for direct extraction of HBT intrinsic elements has been presented. This approach starts with a formulation of the difference between two mutual impedance parameters that can exclude all the extrinsic inductances. In addition, the phase of such an impedance difference has been characterized by several parameters, which establish the fundamentals of the whole extraction. The advantages include higher efficiency and stability in comparison with the other extraction techniques.


## Acknowledgment

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## References


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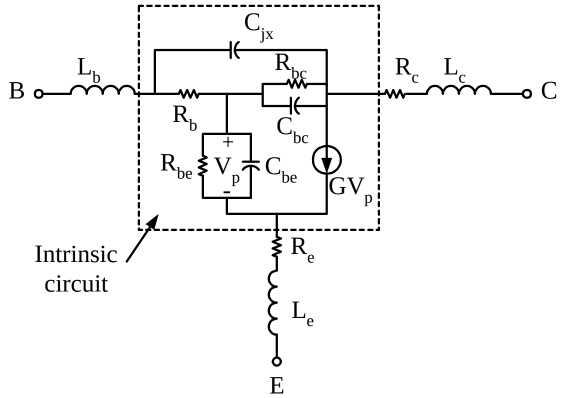


Fig. 1. The hybrid-π small-signal equivalent circuit of the AlGaAs/GaAs HBT.

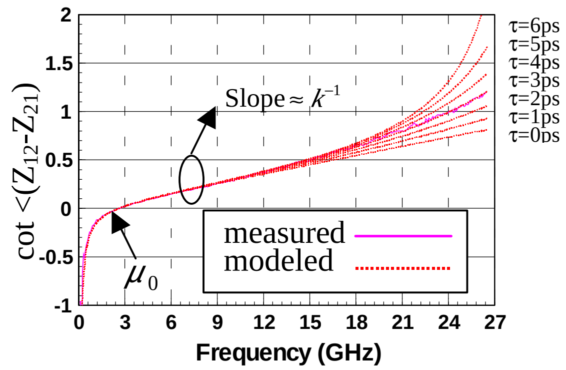


Fig. 2. Curve fit procedure for estimating $\mu_0, k$ and $\tau$.

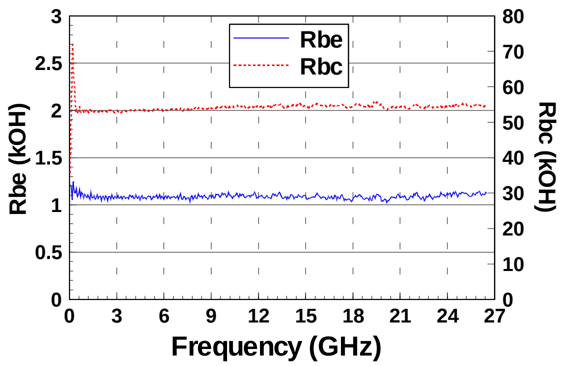


Fig. 3. The extracted quantities of $R_{be}$ and $R_{bc}$.

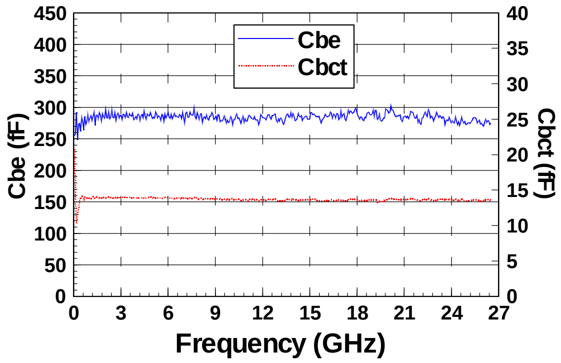


Fig. 4. The extracted quantities of $C_{be}$ and $C_{bc}^{t}$.

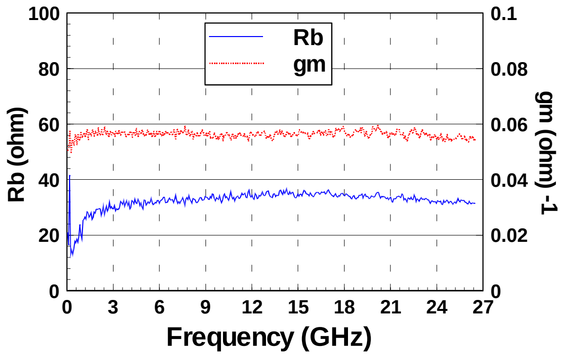


Fig. 5. The extracted quantities of $R_b$ and $g_m$.

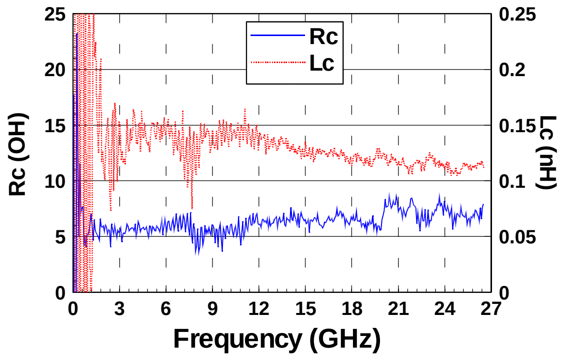


Fig. 6. The extracted quantities of $R_c$ and $L_c$.

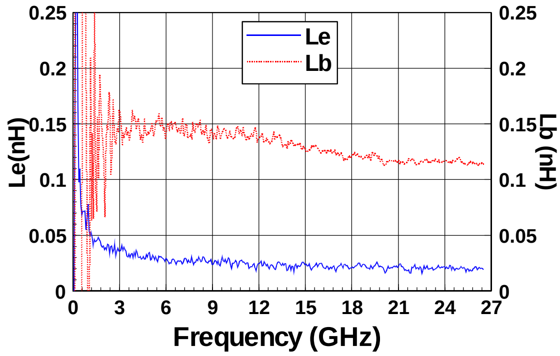


Fig. 7. The extracted quantities of $L_b$ and $L_c$.

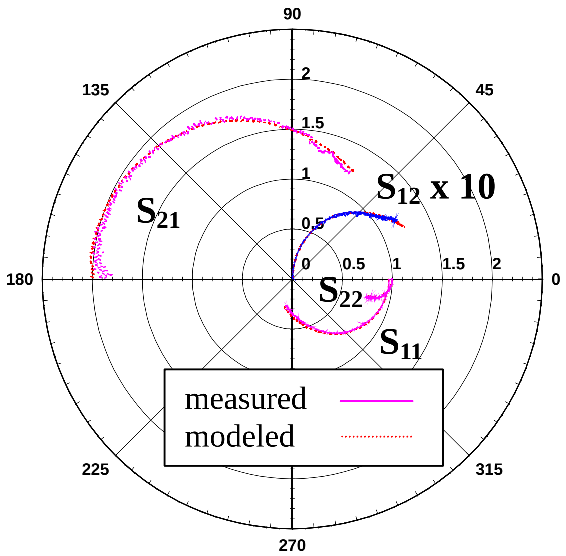


Fig. 8. Comparison between measured and modeled S parameters for the 2.4x3 $um^2$ emitter-area AlGaAs/GaAs HBT operating from 45 MHz to 26.5 GHz.